

Models to Study Birth Intervals and Parity Progression Ratios

Arvind Pandey



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July 2021



(Established in 1956)

बेहतर भविष्य के लिए क्षमता निर्माण

Capacity Building for Better Future

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Models to Study Birth Intervals and Parity Progression Ratios

Arvind Pandey

1. Introduction

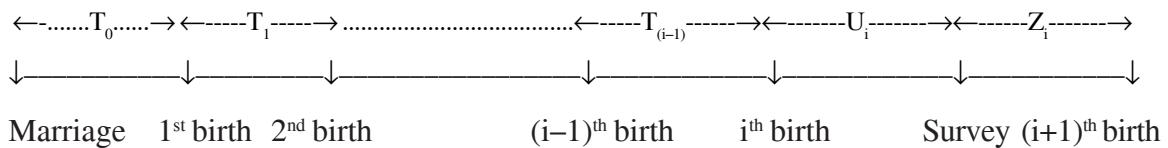
Study of the human reproductive patterns has been of interest to demographers, social scientists and public health researchers to understand the family building process and the life course. For this, data on birth intervals are taken as appropriate as they are the most sensitive measure of human fertility particularly for detecting the current changes in natality patterns of women who are still in their reproductive ages. Conventional measures used for studying fertility do not always provide a clear description of the conditions underlying the observed fertility trends and are not sensitive to small and short-term changes in the reproductive patterns, especially when fertility is highly controlled. In fact, the initial stages of fertility transition occur mainly as a result of earlier cessation of child bearing and afterwards as a consequence of the changes in the spacing between births. The spacing pattern can affect the intrinsic growth rate as well as the mean generational length of the population. Thus, the timing of births can be viewed as a major determinant of population change that can provide insight into the mechanisms underlying the fertility behaviour of a population by disaggregating the reproductive process.

Broadly, human fertility is governed by its two components: (1) the pace with which the women bear children, i.e., the age at which women begin childbearing and have subsequent births; and (2) the degree of transition of women from one specific-parity to the next higher parity, i.e., parity progression ratios (PPR). The former is estimated and analysed based on the birth history data on inter-live birth intervals, whereas the latter requires data on both closed and open birth intervals. There have been efforts to use birth order statistics as well as vital statistics to estimate PPR.

The present paper provides the description of various types of birth intervals, biases and errors in such data, models that takes care of the ascertainment plans and their use in the estimation of the level and determinants of human fertility. It has subsequently discussed about the models of estimating parity progression ratios with illustrative application to NFHS data. It has demonstrated that the use of life table method that combines the closed and open birth intervals can be used to estimate parity progression ratios and intermediate variables affecting fertility with the evolution of multivariate life table method.

1. Types of birth Intervals

Let us have a look at the following sequence of events to visualize the types of birth intervals wherein a woman, after her marriage, has her first conception/live birth, second birth, third birth and so on. Let T_0 be the interval between the consummation of marriage and first birth, T_1 be the time between first and second birth, T_{i-1} be the time between i^{th} and $(i+1)^{\text{th}}$ birth and so on.



From the above we have the following types of birth interval:

- (1) *First birth interval*- the interval between marriage consummation and the first birth is called first birth interval.
- (2) *Inter-live closed birth interval* - the interval between two consecutive births is called inter-live closed birth interval.
- (3) *Open birth interval* - the interval between the date of birth of last child and the survey date is called open birth interval.
- (4) *Straddling birth interval* – any closed birth interval that straddles the survey date is said to be straddling birth interval. In this case, one birth occurs before that age / time point (e.g., survey date) and the other birth occurs after the survey date.
- (5) *Interior inter-live closed birth interval* – the closed birth interval that begins and ends in any segment of time (age-group or the marital duration or interval between two survey dates) is called *interior closed birth interval*.
- (6) *Forward birth interval* - the interval between survey date and the date of next live birth after the survey is called forward birth interval.

2. Biases and errors in the data on birth intervals

The interval between the date of marriage and birth of first child is unique in that it starts with the consummation of marriage and the component of postpartum amenorrhoea which is present in the subsequent birth intervals is absent in this first interval. The first birth interval should be longer than nine months in population where there are no pre-marital relations, while it could be less than nine months, and in a few situations, negative as well, with marriage occurring after the birth of the first child. The values of first birth interval that are less than 7 months (allowing for premature births) and need to be assessed and presented as positive values for analysis.

Further, the data on birth intervals are governed by ascertainment plans (retrospective or prospective or follow-up surveys). Most surveys produce cross-sectional data, where information pertains to experience up to the date of the survey and thereby researchers are confronted with the issues of selectivity, censoring, and truncation. For example, in a sample of recently married women followed month after month to observe the timing of first birth or consecutive births, it was observed that

those who were more fecund conceived sooner while those less fecund conceived later. Thus, the summary statistics based on such sample would be over represented by more fecund women and thereby will exaggerate the rate of conception. This kind of over representation of highly fecund women is called *selection bias*. If everyone had the same conception rate, then the selection effect would not exist. This issue was discussed in detail in a classical paper by Singh et al. (1979a, 1987).

Secondly, in surveys besides inter-live closed birth intervals, a number of incomplete intervals experienced by truncation of women's reproductive history was obtained. These incomplete (open) intervals are called the *censored* (right) observations.

Thirdly, in surveys, there could be non-availability of sufficient exposure period (marital duration) for having at least ' i ' ($i \geq 1$) births. The issue was described in detail based on simulation Sheps et al. (1970), Sheps and Menken, (1972, 1973); Poole (1973); Singh et al. (1979b), Pandey (1981), Pathak and Shastry (1983) etc.

This is more evident from data on mean closed birth intervals from NFHS-3 for the state of Uttar Pradesh in Tables 1-4 for different order and the marriage duration (Yadav and Rai, 2019). The mean closed birth interval between first and second birth was found to be 29 months for women of marriage duration 5-9 years (Table 1), 30.93 months for marriage duration 10-14 years (Table 2), 31.84 months for marriage duration 15-19 years (Table 3) and 32.61 months for women of 15-20 years marriage duration (Table 4). Thus, question arose as to which mean was to be considered as the true mean of the first order closed birth interval, which increased with the increase in marriage duration (exposure period). This was a difficult decision as 29.04 could not be taken as the representative mean birth interval between the first and second birth since women who conceived late and gave birth late, did not have sufficient time to give at least 2 births in a marital duration of 5-9 years. Hence, larger birth intervals could not be included in the study, resulting in lower observed mean.

Table 1: Mean Closed birth intervals (in months) by order for marital duration 5-9 years

No of births	Order of Closed Birth Intervals				
	1	2	3	4	N
2	34.24				679
3	26.32	29.62			494
4	22.57	23.11	26.75		218
5	19.07	20.65	21.82	25.03	71
Total	29.04	26.93	25.52 ^b	25.53 ^c	1470

Table 2: Mean Closed birth intervals (in months) by order for marital duration 10-14 years

No of births	Order of Closed Birth Intervals								
	1	2	3	4	5	6	7	8	N
2	44.15								294
3	31.98	38.88							374
4	28.94	29.42	34.58						361
5	24.28	26.40	26.49	32.82					260
6	22.13	23.29	22.79	24.37	28.46				130
7	20.52	20.98	19.51	23.04	22.15	26.51			55
Total	30.93	29.91	28.91	28.92	25.93	25.68	20.58	17.50	1484

Table 3: Mean closed birth intervals (in months) by order and marital duration 15-19 years

No of births	Order of Closed Birth Intervals								
	1	2	3	4	5	6	7	8	N
2	46.08								175
3	35.29	43.40							271
4	30.13	32.95	38.44						279
5	28.32	31.01	29.26	36.94					268
6	29.22	25.79	27.89	28.65	32.40				223
7	26.93	26.36	23.35	23.18	26.83	30.19			111
8	23.24	22.97	24.80	22.55	23.74	24.23	33.24		74
Total	31.84	32.03	30.35	29.89	28.77	26.89	30.46	23.78	1401

Table 4: Mean closed birth intervals (in months) by order and marital duration 20-24 years

No of births	Order of Closed Birth Intervals								N
	1	2	3	4	5	6	7	8	
2	45.20								175
3	37.03	48.47							271
4	33.43	31.31	39.41						279
5	30.25	30.52	33.45	37.72					268
6	30.75	32.11	30.78	30.84	37.74				223
7	28.78	26.96	27.22	25.56	28.73	36.37			111
8	25.33	26.21	26.08	24.98	26.47	25.49	35.45		74
Total	32.61	33.15	31.31	29.98	30.52	29.95	30.37	32.64	1401

Similarly, the mean closed birth interval between second and third birth was quite small for women with marriage duration 5-9 years (26.93 months) as compared to those of longer marriage duration, viz. 20-24 years (33.15 months). This pattern was observed for other greater order of closed birth intervals as well. This suggested that the chance of including all women who gave two or more births for women of marital duration of 10+ years was almost one, and hence the interval between first and second birth of these women could be considered as reasonable estimates of the mean of the first order closed birth interval. In fact, former (insufficient exposure due to shorter marital duration) was the situation of excluding longer proportion of longer birth intervals. The issue is precisely known to cause *truncation effect* which has been described by Sheps et al. (1970), Pathak and Pandey (1990); Sheps and Menken (1972, 1973).

Further, there is the issue of ascertainment plan which could be demonstrated from the above data given in Table 1 – 4. It may be seen that the mean of the most recent closed birth interval (CBI) was significantly greater than the means of other previous closed birth intervals. For instance, in Table 1, there were 71 women with marriage duration of 5-9 years who had given birth 4 times, and among them the mean of the interval between 1st and 2nd birth, and between 2nd and 3rd birth was about 22 months, whereas the mean interval between 3rd and 4th birth was 25 months. In Table 2, there were 361 women with marriage duration 10-14 years who had 4 births. The mean closed birth interval between the first and second and between the second and third birth was about 29 months while the mean closed birth interval between the third and fourth birth was 34.58 months which was greater than the previous two closed birth intervals of the same group of women, i.e., with 10-14 years of marital duration. Similar trend were observed for the women of the same parity with 15-19

years and 20-24 years of marital duration. Let us take one more example of women of parity 5. There were 260 women who had given 5 births and had a marriage duration of 10-14 years (Table 2). The mean of the most recent closed birth interval (CBI) (32.82 months) was significantly greater than the means of the previous mean closed birth interval (about 26 months). Likewise, from the means of the last closed birth interval of women of greater marital duration, viz., 15-19 (Table 3) and 20-24 years (Table 4) it was observed that the mean of the most recent closed birth interval was significantly greater than the means of other previous CBI. This suggested dealing the two, differently in analysis (Yadav and Rai, 2019). There were various types of errors including the error in recording the date of birth, direct information of the interval between marriage and first birth and subsequent birth intervals and preference of certain digits in such data. Potter (1977) made a detailed analysis of the distribution of births indifferent time intervals prior to the date of survey and for any given duration of marriage. Analyzing the data from World Fertility Survey, Fiji, a U-shaped pattern was observed in reporting of births wherein births appeared to be relatively higher in the period closer to the date of survey and also closer to the date of marriage than expected. This phenomenon was attributed partly to the selective recall lapse and in part to the forward type of questioning adopted in the surveys. Srinivasan (1980), from the same data, subsequently mentioned that in most of the developing countries the distribution of closed birth intervals cluster around multiples of 12 or 6 months, because there was a tendency on the part of the respondents to report the interval between successive births in multiples of whole or half year. If the intervals were reported without any such digit preferences (of twelve or six months) it would be expected that when the intervals were divided by 12 and classified by the residue, the distribution obtained would conform closely to a uniform distribution with a probability of 1/12 at each of the digits 0, 1, 2 ...11. A simple digit preference quotient was computed on the basis of the observed minus the expected values and used for checking the quality of data.

Srinivasan (1980) also discussed the situations where the data on the date of marriage of women, the month of births and in some situations even the year of the occurrence of the events were not known and thereby the data on birth intervals were simply not available. For this, imputations of the month and/or year of the occurrence of the events were made according to some criteria. One simple method of imputing the timing of the birth of a child, where the year of birth was given, it was assumed that the child was born on the first of July of that year, namely, the middle of the year. Though various other procedures for the imputation of the time of occurrence of the events including the choice of a random month within that year existed, Srinivasan (1980) demonstrated his procedure of imputing the time of birth of the child in the middle of the calendar year as simple and statistically viable. However, when imputation was made for two successive births making a birth interval, there were digit preference for such intervals, increasing the digit preference quotient. The proportion of intervals for which an imputation was made (for either of the two births) could itself be used as an index of the quality of data. It was suggested that the extent of digit preference that existed among all the intervals, including the imputed intervals as well as among those intervals excluding the imputed ones, should be considered separately for judging the quality of interval data.

3. Modelling as a suitable approach

These kinds of problems are said to be handled by appropriate models. In fact, modelling is an abstraction of the process indicating the relevant relations among different elements and when expressed mathematically, called mathematical models. Initial efforts to study the birth intervals seem to have been made in French by Gini, Louis Henry, Norman Ryder and some others, and became known when some of the works were translated into English by Mindel Shepsin the early seventies (Henry, 1972). The first model appears to have been on the time of first conception.

3.1 Time to first conception

If 'p' was the monthly chance of conception (fecundability) and conception was the outcome of a sequence of Bernoulli trials, the time of first conception after marriage consummation (X) followed a geometric distribution, i.e.,

$$P(X = x) = q^{x-1}p, \quad x = 1, 2, \dots, \quad 0 < p < 1, \quad q = 1 - p. \quad \dots (1)$$

$$E(X) = \sum_{x=1}^n x q^{x-1} p = \frac{1}{p}; \quad \text{and} \quad \dots (2)$$

$$\text{Var}(X) = \frac{q}{p^2} \quad \dots (3)$$

Hence, 'p' could be estimated from the data on time to first conception/birth. If the population of women was heterogeneous with respect to fecundability p, then,

$$P(X = x) = \int q^{x-1} p f(p) dp \quad \dots (4)$$

Potter & Parker (1964) and Sheps (1964) assumed f(p) to be Beta distribution.

If time was treated as a continuous random variable and 'm' was the conception rate, the probability model of T_0 was given by,

$$f_0(x) = m e^{-mx} \text{ for homogeneous population of women,} \\ = \int m e^{-mx} f(m) dm \text{ for heterogeneous population of women.} \quad \dots (5)$$

Singh (1964) considered f(m) to be a type III Gamma distribution.

Note that the above models were under the assumption that all women under study were observed for the outcome for a long time which was not always feasible. Hence, Suchindran and Lachenbruch (1974) and a few others developed truncated version of the model and estimated the parameters.

Efforts were also made to incorporate the chance of foetal losses before the occurrence of the first live birth and methods to estimate the parameters from the truncated data. Pathak and Prasad

(1977) included and estimated the extent of adolescent sub-fecundity/infertility for the population where consummation of marriage used to take place early before the maturation of couples.

3.2 Modelling inter-live birth intervals

The model was further extended to analyze inter-live birth intervals by Srinivasan (1966) by incorporating the period of post-partum amenorrhoea (PPA) after a birth.

Accordingly, probability density function $f_i(x)$ the inter-live birth interval T_i was:

$$f_i(x) = m e^{-m(x-h)} \text{ for homogeneous population of women,} \quad \dots (6)$$

$$= \int m e^{m(x-h)} f(m) dm \text{ for heterogeneous population of women.} \quad \dots (7)$$

Where " m " was the conception rate and h = gestation period + PPA.

The discrete-time model of (6) was first used by Srinivasan (1966) while Suchindran and Lachenbruch (1974) used the continuous time model (7) to study the birth intervals. Several researchers analyzed birth interval data in different parts of the world, for example, Jain (1969), analyzed data on birth intervals from Taiwan; Chakraborty (1974) from Varanasi Survey etc. However, they did not visualize the kind of problems stated above. It was Sheps et al. (1970) who first visualized the problem of truncation and distinguished the difference between birth intervals of those women who had given at least $(i+1)$, and those who had given exactly " $(i+1)$ " births, prior to the survey.

The model of the interval between i^{th} and $(i+1)^{\text{th}}$ birth for women who had given at least $(i+1)$ birth in $(0, T)$, was given by,

$$h_i(x/T) = \frac{G_i(T-x) f_i(x)}{G_{i+1}(T)}, \quad i \geq 0, x \leq T \quad \dots (8)$$

Where $G_i(T)$ was the waiting time distribution function for i^{th} birth.

The model for the interval between i^{th} and $(i+1)^{\text{th}}$ birth for women who had given exactly $(i+1)$ birth in $(0, T)$ was given by,

$$h_{iL}(x/T) = \frac{P_i(T-x) f_i(x)}{P_{i+1}(\bar{l})}, \quad i \geq 0, x \leq T \quad \dots (9)$$

Where, $P_i(T)$ was the probability distribution of " i " births and given by the difference of two consecutive distribution functions, i.e.,

$$P_i(T) = G_i(t) - G_{i+1}(t), i > 0. \quad \dots (10)$$

Note that the above model remained inside the simulation laboratories and was appreciated by limited researchers.

By taking $f_0(x)$ and $f_i(x)$ as (5) and (6) and one-to-one correspondence between a conception and a birth, $G_i(T)$ and $P_i(T)$ were given by,

$$G_i(t) = 1 - e^{-m(T-(i-1)h)} \sum_{j=0}^{i-1} \frac{[m(T - (i-1)h)]^j}{j!} \text{ and} \quad \dots (11)$$

$$P_0(T) = 1 - \alpha e^{-mT}; \quad \dots (12)$$

$$P_i(T) = \alpha [e^{-m(T-ih)} - \sum_{j=0}^{i-1} \frac{[m(T - ih)]^j}{j!}] \quad \dots (13)$$

Where, α was the probability that the woman under study was susceptible.

Hence, the corresponding mean and variances of X_i and X_{iL} could be derived.

With the help of these models, the data on birth intervals could be analysed to get reliable estimates of the parameters that account for the ascertainment plan and *truncation effects*. While Indian researchers like Singh et al. (1979b), Pathak and Shastry (1983); Bhattacharya et al. (1988) and Pandey et al. (1988; 1990). propounded these models with application to sample survey data, Yadav and Rai (2019) had reviewed and analysed birth intervals with application to NFHS data and estimated various parameters. It was concluded that it was preferable to analyse the most recent closed birth intervals which were relatively easy to ascertain and correlate to the current fertility of women, although by using a proper method (right model).

3.3 Open birth interval

In retrospective surveys, besides inter-live closed birth intervals, we have the interval between the date of birth of the last child and the survey date which is called as open birth interval Srinivasan (1967a, 1967b, 1968). The data on closed and open birth interval may be treated as complete and censored and combined into one under the life table framework. The net effect of the variables on the timing of births could be examined through the development of multivariate life table (Cox, 1972; Rodriguez and Hobcraft, 1980; Rutherford and Choe, 1993 etc.). The basic model was the proportional hazard model which expressed the logarithm of the hazard rates as a linear function of a set of independent variables or covariates as follows:

$$\lambda(t, x) = \lambda_0(t) e^{\beta x} \quad \dots (14)$$

Where, $\lambda(t, x)$ was the hazard rate at time t , $\lambda_0(t)$ was the baseline hazard function, X was a vector of covariates and β was a vector of the corresponding regression coefficients. The baseline hazard was like a constant term in ordinary least square equation. For a unit increase in a given covariate X , the multiplicative change in the baseline hazard rate was given by $\exp(\beta)$. It referred to the instantaneous rate of experiencing an event when all the covariates take the value of zero. Since the

event of interest was the waiting time to experience birth of a particular order, $\lambda(t, x)$ was a time specific hazard rate in a small interval of time t for a covariate X . In subsequent researches, time dependent covariates were also followed (Rutherford and Choe, 1993).

Though such reforms in life table functions made it possible to analyze data on birth intervals under a multivariate analysis, it required appropriate models to handle truncation effect and ascertainment plans. Hence, Sheps et al. (1970) presented a general model of open birth interval of women of parity “ i ”, $k_i(u|T)$, as follows:

$$k_i(u|T) = \frac{g_{i-1}(T-u) Q_i(u)}{P_i(T)}, \quad \dots (15)$$

where, $g_{i-1}(T-u)$ was the cumulated time probability density function, $Q_i(u)$ was the probability of “no birth” in time interval of length “ u ” after the i^{th} birth, and $P_i(t)$ was the probability of exactly “ i ” births in time $(0, T)$.

For large T (marriage duration), the model was as simple as:

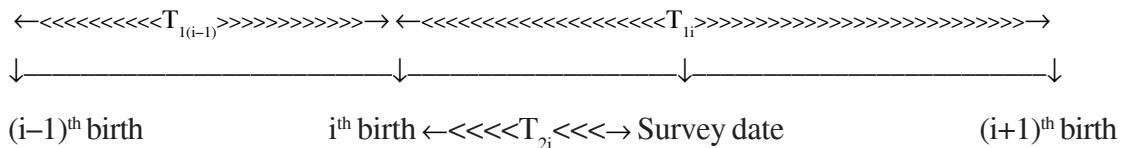
$$u_i(x) = \frac{Q(x)}{E(T)} = \frac{1-F(x)}{E(T)} \quad \dots (16)$$

where, $Q(x)$ was the probability of “no birth” in time interval of length “ u ” after the last birth, and $E(t)$ was the mean inter-live closed birth interval for the steady state. Pathak (1971), Singh et al. (1982) and Pandey and Pathak (1989) derived explicit models of open birth interval and estimated fertility parameters, fecundability and sterility.

Utilizing the data from World Fertility Survey of Fiji, Srinivasan (1980) carried out a correlation analysis and used a simple regression model to estimate the total marital fertility rate (TMFR) from the age-specific average open birth interval. The fitted regression line provided reasonably good estimates of TMFR for different situations. Yadava and Rai (2019) analyzed fitted a regression model with TFR as the dependent variable and proportion of women having open birth interval more than 5 years among married women in the age group 15–49 years, as the independent variable. It used the data of NFHS-3 and gave reasonably satisfactory fit (R^2 values more than 0.9) and the estimates of TFR for different states of India and different groups of the population. It would certainly be useful if the proposed procedure was applied to other sets of data to check the adequacy of these procedures thereby providing further confidence on the application of these procedures to draw meaningful inferences.

3.4 Straddling Birth Interval

In the following figure where Leridon (1969) suggested that the interval T_{li} which included the survey date was proportional to the length of T_{li} , i.e., larger intervals of T_{li} had greater chances of inclusion of a survey than smaller intervals and it was a straddling closed birth interval.



Hence, the model for straddling birth interval, $k_i(t)$, would be proportional to the length and distribution of usual closed birth intervals, i.e., $k_i(t) \propto t f_i(t)$ and hence,

$$k_i(t) = \frac{t f_i(t)}{E(T)} \quad \dots (17)$$

where $E(T)$ was the mean of $T_{1(i-1)}$. The same was earlier found by Henry (1953, 1972). Specific models were developed by Yadav and Pandey (1989) and used with real data to estimate the parameters. This feature is also called as length biased sampling because the intervals of larger lengths had larger probability of inclusion in the study.

3.5 Interior Birth Interval

Earlier it was described that the interior birth interval was the closed birth interval that began and ended in any segment of time (which may be the age group or marital duration or interval between two survey dates). The above interval was not only dependent on the fertility parameters involved in the model but also upon the condition of the occurrences of birth in a given age interval/marital duration. All these made the distribution quite complex and perhaps due to this reason not much work was done on this birth interval. However, some researchers derived probability distributions of interior birth intervals under varying sets of assumptions (Bhattacharya and Singh, 1986; Pandey et al. 1987).

3.6 Forward Birth Interval

As mentioned earlier, a forward birth interval is defined as the time between survey date and the next birth posterior to the survey date. It is similar to forward recurrence time in renewal process. Open birth interval may be treated as backward recurrence time. In retrospective studies data on open birth interval could be obtained easily and utilized for drawing inferences about fertility behaviour of women. However, to obtain data on forward birth interval a prospective study is needed. However, in some situations forward birth interval might be useful in fertility studies.

In the renewal theory, the limiting forms of the probability density functions of backward recurrence time and forward recurrence time were identical on the assumption that renewal densities did not change over time. However, if renewal density changed after some time (say survey point), then the distribution of open birth interval and forward birth interval would not be identical. Such a situation could occur if a certain family planning program was introduced at the time of survey such that the fertility parameter, viz., conception rate, changed after survey. It is desirable to measure this change

so that adequacy of the family planning program may be assessed by comparing the distribution of the open birth interval and forward birth interval. In this context, Singh et al. (1978) derived a probability model for forward birth interval under the assumption that a family planning program had been introduced in the population at the survey point. They also obtained expressions for mean and variance under different situations.

4. Parity progression ratios

French demographers Luis Henry (1953) and Norman Ryder (1953), as discussed in Ryder (1986), introduced the concept of parity progression ratios (PPR) as the probability that a woman after delivering her i^{th} birth will ever proceed to the next birth (p_i).

$$p_i = \frac{\text{Number of women who have } (i+1) \text{ birth}}{\text{Number of women who have } i \text{ birth}}, i \geq 0. \quad \dots (18)$$

It is a conditional probability, and the cumulative probability would give the estimate of total fertility rate:

$$\text{TFR} = p_B p_0 + p_B p_0 p_1 + p_0 p_1 p_2 + p_0 p_1 p_2 p_3 + \dots + p_B p_0 p_1 p_2 p_3 p_4 p_5 p_6 / (1-p_6) \quad \dots (19)$$

Where p_B = probability of ever marriage in the population and for $p_B = 1$,

$$\text{TFR} = p_0 + p_0 p_1 + p_0 p_1 p_2 + p_0 p_1 p_2 p_3 + \dots \quad \dots (20)$$

This concept, however, did not gain wide application due to issues related to its measurement, data needs, and conceptualization with respect to cohort and period measures. Having introduced the concept of open birth interval, Srinivasan, in a series of papers (Srinivasan, 1967a, 1967b, 1968), propounded the method of estimating instantaneous parity progression ratios from such data. A brief description of the method is provided in the following sections.

4.1 Closed and open birth interval as a feasible data

Let us recall the model of open birth interval (16):

$$u_i(x) = \frac{Q(x)}{E(T)} = \frac{1-F(x)}{E(T)}$$

From this, Srinivasan (1967a, 1967b, 1968) presented that mean and second moment of open birth interval, i.e., the time between i^{th} birth, which is the last birth and the survey date (U_i) as:

$$E(U_i) = \frac{E(T_i^2)}{2E(T_i)} \quad \dots (21)$$

Where, $E(U_i)$ is the mean of open birth interval and, $E(T_i)$ and $E(T_i^2)$ are mean and second moment of T_i (inter-live birth interval between i^{th} and $(i+1)^{\text{th}}$ birth).

Note that the above results were true only for those women who continued to reproduce at least up to $(i+1)^{th}$ birth.

For those women who gave birth to their i^{th} child between “u” time prior to the survey (open birth interval) and that i^{th} birth happened to be the last birth, the mean open birth interval was:

$$E(U_i) = \frac{E(V_i^2)}{2E(V_i)} \quad \dots (22)$$

Where, V_i was the interval between the date of birth of last child (for parity “i” women) and end of reproductive age, say 45 years.

If α_i was the proportion of women who went for the next higher order $(i+1)^{th}$ birth and $(1-\alpha_i)$ did not go for the next birth, the final expression of mean open birth interval according to Srinivasan (1967a, 1968) would be:

$$E(U_i) = \alpha_i \frac{E(T_i^2)}{2E(T_i)} + (1-\alpha_i) \frac{E(V_i^2)}{2E(V_i)} \quad \dots (23)$$

With the knowledge of mean open birth interval, $E(U_i)$, $E(T_i)$, $E(V_i)$ and $E(V_i^2)$, an estimate of α_i can be made Srinivasan (1967a, 1968) termed this α_i the *instantaneous parity progression ratio* ($IPPR_{i \rightarrow i+1}$).

Following this, two questions arose: what would be the parity progression ratio (PPR) and where to get data on V_i ? Data on V_i were seldom available in fertility surveys and even if available, they suffered from various types of biases. In this context, Yadav and Saxena (1989) and Yadav et al. (1992, 2013) addressed both the issues and considered only those OBI which were less than a specified value, say, C such that $P[T_i \geq C] \approx 0$.

Let α_i^* be the probability that the woman after i^{th} birth went for the next higher order $(i+1)^{th}$ birth and $(1 - \alpha_i^*)$ did not go for the next birth. Considering only those OBI which were less than C, the total number of fecund women having $OBI < C$ would be:

$$\int_0^C \alpha_i^* B_i [1 - F_i(u)] du = \alpha_i^* B_i E(T_i) \text{ and} \quad \dots (24)$$

the total number of infecund women with $OBI < C$ would be:

$$\int_0^C (1 - \alpha_i^*) B_i du = (1 - \alpha_i^*) B_i C. \quad \dots (25)$$

So, the total number of women at the time of survey with $OBI < C$ would be

$$\alpha_i^* B_i E(T_i) + (1 - \alpha_i^*) B_i C.$$

Thus, the proportion of women in the sample at the time of the survey with $OBI < C$ who would go for the next higher order $(i+1)^{th}$:

$$\alpha_i^* = \frac{\alpha_i^* E(T_i)}{\alpha_i^* E(T_i) + (1-\alpha_i^*)C} \quad \dots (26)$$

The mean OBI of women whose $OBI < C$ was given by,

$$E(U_i^C) = \alpha_i \frac{E(T_i^2)}{2E(T_i)} + (1 - \alpha_i) \frac{C}{2} \quad \dots (27)$$

After solving the above, the following was obtained

$$\alpha_i^* = \frac{C^2 - 2CE(U_i^C)}{C^2 - 2[E(T_i^2)] - \{C - E(T_i)\} E(U_i^C)} \quad \dots (28)$$

Thus, with the knowledge of $E(U_i^C)$, $E(T_i)$, $E(T_i^2)$, and C , both α_i (IPPR) as well as α_i^* (PPR) could be estimated.

The above concept of introducing C (a specified value) and using mean of open birth interval truncated at C , could be tried under the life table framework as well.

4.2 Life table combining closed and open birth intervals to estimate parity progression ratios

The parity transition was the proportion of women who had given birth within a given period, that is, the first transition from a woman's own consummation of marriage to the occurrence of her first order birth, and from first birth to second order birth and so on. In general, the starting event was the consummation of marriage or a birth of a particular order, say, i^{th} birth and next event was $(i+1)^{\text{th}}$ order birth or the survey date.

The life table approach discussed earlier in 4.3 suggested combining the last closed and open birth intervals into a single variable of the time to event. Here we define the time interval $(t_{ij} \leq t < t_{ij+1})$ during which an individual woman, having had i^{th} birth, either experience her $(i+1)^{\text{th}}$ birth or do not experience a birth. In latter situation, the interval is open birth interval and called censored times for the specific i^{th} birth. Suppose that d_{ij} and m_{ij} , respectively, were the number of births (event) and censored (failures) observations during the interval and N_{ij} was the number of women at the start of the interval with specific parity i . Define $n_{ij} = N_{ij} - \frac{m_{ij}}{2}$ as the adjusted number at risk at the start of the interval.

The product-limit estimate of the conditional survivor function is:

$$S_{ij} = \prod_{k=1}^i \frac{n_{ik} - d_{ik}}{n_{ik}} \quad \dots (29)$$

Then, $1 - S_{ic}$, would approximate as the parity progression ratio p_i for a priori value of $T_{ij} = C_i$. Based on all India data on birth intervals in the third round of National Family Health Survey,

2005-06 (NFHS-3), the mean and second moment of T_i (closed birth interval between i^{th} and $(i+1)^{\text{th}}$) and U_i for all ever married women aged 15-49 years are presented respectively for first four columns of Table 5. The estimates of instantaneous parity progression ration (IPPR denoted by α_i) based on model (26), parity progression ratios (PPR denoted by α_i^*) from model (28) and life table-based Kaplan Meir parity progression ratios (PPR denoted by $KM\alpha_i^*$) and given in model (29) are presented in last three columns of the Table 5. Based on these estimates of parity progression ratios and substituting them into the model equation (19), implied total marital fertility rate (TMFR) was derived as provided in the last row of the Table 5. As expected, the instantaneous parity progression ratios (IPPR, i.e., α_i) were consistently smaller than the parity progression ratios (PPR, i.e., α_i^*). Further, the PPR based on life table denoted by $KM\alpha_i^*$ were greater than the estimates based on direct use of mean closed and open birth intervals for higher parities, say after parity 2.

Table 5: Estimates of PPR & IPPR for ever-married women aged 15-49 years NFHS-3, India (2005-06)

Parity	$E(T_i)$	$E(T_i^2)$	$E(U_i)$	$E(U_i^2)$	α_i	α_i^*	$KM\alpha_i^*$
P_{0-1}	27.6	1207.56	28.2	1578.99	0.83	0.96	0.96
P_{1-2}	36.8	1740.78	30.8	1717.07	0.80	0.93	0.94
P_{2-3}	35.4	1610.36	44.6	3118.75	0.41	0.70	0.76
P_{3-4}	34.4	1514.91	50.9	3839.1	0.24	0.52	0.67
P_{4-5}	34.1	1471.74	51.8	3900.06	0.21	0.49	0.65
P_{5-6}	33.4	1432.35	52.6	3995.63	0.19	0.46	0.63
P_{6+}	32.6	1336.75	52.0	3967.01	0.20	0.48	0.63
Implied TFR for $p_B=0.96$					1.8	3.0	3.5

However, the implied TMFR based on the life table as 3.5 was quite close to what was observed (3.6) in the survey (IIPS and Macro International, 2007).

Further, this life table can be extended to multivariate framework as described in (14) as follows,

$$p_i(c,x) = 1 - S(c;x) \quad \dots (30)$$

This was the cumulative survival at a fixed value c - gave the estimates of PPR (Gandotra et al. 1999, Retherford et al., 2010; 2013; Narayan et al., 2017, 2020) corresponding to specific covariate. One can MCA over the estimates of relative position of $1 - S(c;x)$ provided in Table 6, to estimate PPR in different sub-groups of the population under study. Nonetheless, it can be inferred that while progression from marriage to first birth in urban area was greater than in rural areas, the parity progression ratio was consistently smaller after second birth, viz, from first to second birth,

third to fourth birth and fourth to fifth birth in urban areas than that in rural areas. This eventually led to smaller TMFR in urban areas than in rural areas. A similar pattern was observed based upon the mother's education. Greater the education level lower was the progression from marriage to first birth possibly due to sub-fecundity on account of early age at marriage amongst those educated less than primary. The parity progression after parity 2 was significantly lower among educated women leading to smaller total fertility rate.

Table 6: Estimated relative position of 1-S(c; x) by place of residence, India, NFHS-3 (2005-06)					
Parity →	0-1	1-2	2-3	3-4	4-5
Place of residence					
Rural	1.00	1.00	1.00 for	1.00	1.00
Urban	1.14*	0.80*	0.71**	0.74**	0.78**
Mother's education					
<Primary	1.00	1.00	1.00	1.00	1.00
Primary	1.08*	0.95*	0.72**	0.65**	0.67**
Secondary	1.20*	0.78**	0.45**	0.45**	0.48**
Higher	1.23*	0.45**	0.17**	0.21**	0.29**

*Significant at 5% level of significance, ** Significant at 1% level of significance
Source: Part of ongoing work of my students Padum Narayan (2021) Ph.D. thesis.

5. Modelling family building process with age-parity-specific force of fertility

Models have been developed to use age-parity-specific fertility rates to derive various fertility measures including the mean maternal age at different order of births, birth intervals and parity progression ratios (Das Gupta & Long, 1985; Pandey and Suchindran, 1997). While details could be found in those papers, a brief of the same is presented here for immediate exposition.

Let $m_i(a)$ be the age-parity-specific force of fertility such that a woman of age 'a' and parity 'i' will give birth in a small age interval 'a' and 'a+da'. Let α and β , respectively, denote the lower and upper age limits of reproductive life. The distribution of maternal age at i^{th} birth, $f_i(x)$ would be given by,

$$f_i(x) = \int_{\alpha}^{x} m_i(y) e^{-\int_y^x m_i(a) da} f_{i-1}(y) dy \quad \dots (31)$$

with distribution function,

$$F_i = \int_{\alpha}^{\beta} f_i(x) dx \quad \dots (32)$$

Hence, the proper distribution of the age at i^{th} birth, say, $f_i^*(x)$ was given by,

$$f_i^*(x) = \frac{f_i(x)}{F_i} \quad \dots (33)$$

The corresponding mean maternal age at i^{th} birth could be easily derived as,

$$\bar{X}_i = \int_{\alpha}^{\beta} x f_i^*(x) dx. \quad \dots (34)$$

Similarly, the distribution of the interval between $(i-1)^{\text{th}}$ and i^{th} birth for all those women whose $(i-1)^{\text{th}}$ took place at age z , was given by

$$g_i(u|z) = m_i(z+u) e^{-\int_z^{z+u} m_i(t) dt}$$

The unconditional distribution of U_i was given by

$$g_i(u) = \int_{\alpha}^{\beta-u} g_i(u|z) f_{i-1}(z) dz.$$

Hence, the proper distribution of U_i was:

$$g_i^*(u) = \frac{1}{F_i} g_i(u). \quad \dots (35)$$

The mean interval between i^{th} and $(i+1)^{\text{th}}$ birth, denoted by \bar{U}_i might be derived as:

$$\bar{U}_i = \int_{\alpha}^{\beta} u g_i^*(u) du. \quad \dots (36)$$

$$\text{The ratio } \frac{F_{i+1}}{F_i} \quad \dots (37)$$

gave the estimate of parity progression ratio (p_i).

Chiang and van den Berg (1982), with the knowledge of mean maternal age at different order of birth, mean birth intervals and parity progression ratios, suggested constructing a fertility table of life as follows:

Considering that there was a cohort of l_0 women of parity zero and average age \bar{X}_0 to be observed for their entire reproductive life, let l_t be the number of women of the original cohort of l_0 women

who experience i or more births. It was equivalent to the number of women of parity “ i ” eligible to have the next $(i+1)^{th}$ birth and could be expressed recursively as:

$$l_{i+1} = l_i p_i, i=0, 1, 2, \dots, k-1. \quad \dots (38)$$

The analogy to ${}_n L_x$ column in the life table was given by

$$L_i = l_{i+1} \bar{U}_i \quad \dots (39)$$

$$\text{And } T_i = \sum_{i=1}^k L_i \quad \dots (40)$$

Hence, the mean age of reproductive life of women at parity “ i ” was given by

$$e_i = \frac{T_i}{l_i} \quad \dots (41)$$

Hence, the mean age at last birth for a woman of parity i could be estimated as the sum of the mean age at the i^{th} birth and e_i .

Thus, armed with knowledge of age-parity-specific fertility intensity, the model expressions described above may be used to estimate the mean age at different births, birth intervals, and parity progression ratios. For application, age-parity-specific fertility intensity might be approximated as age-parity-specific fertility rates. Further, the models were derived on a continuous time scale whereas data were available only on discrete time scale. Therefore, there was a need to approximate the integrals in the models by summations over the age of the woman in the reproductive life span. Besides, the fertility rates needed to be considered on a single year age instead of the conventional five-year age vital rates. Based on the data on age-parity-specific fertility rates in India as per Census 1981, Rajaram (1996) estimated the mean age at different births, birth intervals and parity progression ratios and constructed the fertility table of life reproduced in Table 7 from the said work for rural India.

Table 7: Parity-specific fertility table derived from Age-parity-specific fertility rates for rural India

Parity	l_x	p_i	\bar{X}_i	\bar{U}_i	L_i	T_i	e_i	Expected age at completion of family
0	100000	0.9842	16.5	3.51	345454	1868843	18.68843	35.19
1	98420	0.9349	20.01	4.53	416818	1523389	15.47844	35.49
2	92013	0.8756	24.08	4.37	352075	1106570	12.02626	36.11
3	80566	0.8011	27.53	4.36	281402	754495	9.364876	36.89
4	64542	0.7275	30.43	4.27	200494	473093	7.330021	37.76
5	46954	0.6573	32.99	4.17	77760	77760	2.519514	37.45
6+	30863	0.6042	34.93	—	—	—	—	—

Source: Rajaram (1996), Ph.D. thesis at IIPS, Mumbai.

The application of such models to derive fertility tables for several countries were made by Bhrolchain (1987), Feeney and Yu (1987) and Lutz and Fitchimger (1985). Giving the argument that age and parity were strongly related and data on age-specific rates were more frequently available, Krishnamoorthy (1979) and Pandey (1995, 2002) presented simplified versions of the above models to estimate the mean maternal age at births of different order, birth intervals and parity progression ratios. Subsequently, Sreenivasan (2002) used NFHS-1 data examining the impact of including the period of non-susceptibility (PPA) in the model. One can also see the simulation exercise undertaken by Venkatcharya and Roy (1972) to estimate fecundability from age-specific rates and derive other measures of fertility.

6. Use of birth order statistics

Blacker et al. (1989) reviewed and applied an unpublished technique of Brass (1975) to estimate fertility as well as the parity progression ratios when the number of women in different age groups were unknown. It assumed that all the births of each order occurred at the mean age of the mothers. If \bar{X}_i was the mean age of mothers at the birth of the i^{th} child and \bar{X} , the mean age of mothers at all births, the relative difference in age of mothers at births of different orders became $(\bar{X}_{i+1} - \bar{X}_i)/\bar{X}$, which could be denoted as k_i . The number of women declined exponentially with age and the rate of decline was a function of the mean reproduction rate. The formula that reweighed the number of births at different orders was as follows.

$$F_i = B_i (Gl_1)^{k_i} \quad \dots (43)$$

where, F_i and B_i were, respectively, the weighted and reported number of births of order “ i ”, G was the gross reproduction rate (GRR) in the population, l_1 was the life table survivors at age 1 such that $(1-l_1)$ was the infant mortality rate. The ratio of F_{i+1} and F_i was thereafter taken to provide an estimate of PPR.

Yadava and Srivastava (1994) modified the above model which did not require data on GRR and l_1 in the population under study. The procedure was based on the argument that if the number of women at all ages in the reproductive period were the same, then the number of births of order “ i ” and order “ $i+1$ ” would be equal only if PPR corresponding to the parity was one. However, in a real situation, the number of women at all ages were not equal and hence, even if PPR was one, the number of births of order “ i ” might be larger than the number of births of order “ $i+1$ ” in a year simply because of mortality among women after the i^{th} birth. Some adjustment factor was, therefore, required to account for this phenomenon. This adjustment could be made by adding $B_i (\bar{X}_{i+1} - \bar{X}_i) r$ into the number of births of order $(i+1)$, i.e., B_{i+1} where, r represented the rate of growth of the population, \bar{X}_i and \bar{X}_{i+1} had the same meaning as above. Accordingly, if B_{i+1}^* denoted the adjusted number of births of order $(i+1)$, it was given by:

$$B_{i+1}^* = B_{i+1} + B_i (\bar{X}_{i+1} - \bar{X}_i) r \quad \dots (44)$$

The ratio of B_{i+1}^*/B_i gave an estimate of parity progression ratios corresponding to parity "i".

Pandey (1994) analyzed the data on birth order statistics (B_i) from Census (1981) and Sample Registration System (1990). They first estimated \bar{X}_i using model expression (34) described in the present study. With the knowledge of GRR, IMR and growth rates of the population they used models (43) and (44) and estimated the parity progression ratios as presented in Table 8. The estimates of PPR obtained by these two procedures were the same at least at the lower parities. It may be mentioned that the estimates of PPR were more sensitive to changes in the distribution of births (B_i) than other inputs, i.e., the mean age at different order of births, GRR and IMR required in these models. In addition, Ram and Pathak (1989) and Srinivasan et al. (1992) also made use of birth order statistics to estimate PPR in India.

Table 8: Birth order distribution, India, Census (1981) and estimates of parity progression ratios							
Parity	Number of birth (B_i)	\bar{X}_i	K_i	F_i	PPR Model (43)	B_i^*	PPR Model (44)
1	4285507	19.9		4285507		4285507	
2	3581936	24.0	0.1390	3939333	0.9192	3970717	0.9265
3	2923695	27.4	0.2574	3486765	0.8851	3200507	0.8935
4	2258591	30.3	0.3559	2881367	0.8264	2446550	0.8368
5	1566024	32.9	0.4451	2123616	0.7370	1697564	0.7516
6	1065970	34.8	0.5113	1512538	0.7122	1133672	0.7239
7	685334	36.4	0.5673	1010386	0.6680	734255	0.6794

The value GRR=2.24; $l_1 = 80.885$, $r=0.0224$, Source: Pandey et al. (1994)

7. Summary and conclusion

The present treatise has described and discussed the usefulness of birth intervals including the interval between marriage consummation and first birth, between subsequent reproductive events (births) in the life cycle of women to examine the level, trends and determinants of human reproductive behaviours of the population. Data on birth intervals was also useful for evaluating the impact of a programme and provide future directions. Various errors and biases which could be present due to the incomplete reproductive history of women in retrospective surveys are discussed. The paper has described the analytical methods for checking the quality of data on birth intervals reported in retrospective surveys and methods of adjustments for defective or incomplete data in such variables as per the existing literature.

The analysis of such data has been in vogue for quite some time but most of the research was in French. In the beginning, data on the time to first birth was analysed to with the help of models to estimate the monthly chance of conception (fecundability). There were generalizations of the model to incorporate the provision of individual variability to fine tune the estimate of fecundability. Models were extended to estimate the extent of sub-fecundity for the population where there were practices of early marriage and the partners were not mature enough for conception in certain parts of India. The above model of the first birth interval was extended with the inclusion of the duration of post-partum amenorrhoea to analyse the interval between two births in the sixties. However, when it was realized that the ascertainment plans could affect the distribution of birth intervals, models were developed for various types of birth intervals for specific marriage duration. They also dealt with issues of truncation, censoring and selectivity in the data and analysis of birth intervals through appropriate models as discussed in the manuscript.

A review of development of the model was included to analyze birth intervals and estimation of parity progression ratios. The recently distinguished instantaneous parity progression ratios (IPPR) and parity progression ratios (PPR) were also reviewed. It was demonstrated that the life table method could be used not only to analyze birth intervals but also to estimate parity progression ratios. One could study the factors affecting reproductive timing and parity progression ratios under multivariate life table framework.

Also reviewed were efforts to estimate reproductive timings and parity progression ratios from vital statistics, viz. age-parity-specific fertility rates and sometimes only age-specific fertility rates with illustrative examples to construct a fertility table of life describing the entire reproductive span in a synthetic cohort. In conclusion, use of birth order statistics to estimate parity progression ratios were delineated with illustrative data from literature. The models and method described above could be applied with the new and time series data examining the levels and trends of fertility and impact of various programmes of family planning and development.

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About the Author

Prof. Arvind Pandey, a veteran in the field of statistical modelling, population science and epidemiological studies obtained his Ph.D. degree in Statistics from the prestigious Banaras Hindu University in 1981. He started his career as Lecturer at IIPS, Mumbai, in 1982 and was elevated to the post of Reader in 1989 and Professor in 1998. He later joined ICMR - National Institute of Medical Statistics (NIMS) in mid-2000 as Director and continued till January 2017. He also worked as the Advisor to DG, ICMR and National Consultant (Statistics) to WHO before joining the position of National Chair (Medical Statistics) at ICMR in November 2019. He coordinated part of the first and second round of the NFHS and has been guiding as the Chairman of the Technical Advisory Committee (TAC) for NFHS-3 (2005-06), Member for NFHS-4 (2015-16) and Co-chair for NFHS-5 (2018-19). His area of teaching and research included demographic models, survey methodology, estimation and projection of HIV/AIDS and clinical biostatistics. He has authored over 230 peer-reviewed articles, 12 books and 45 research monographs/reports, and 18 students have been awarded Ph.D. under his supervision/co-supervision. Prof. Pandey is presently a member of the India's National Task Force on COVID-19 - Operation Research Group and National Disease-Free Certification for Tuberculosis.

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